

ON THE EQUIVALENT CIRCUIT AND PERFORMANCE OF PLATED QUARTZ BARS

Also
IN THIS ISSUE

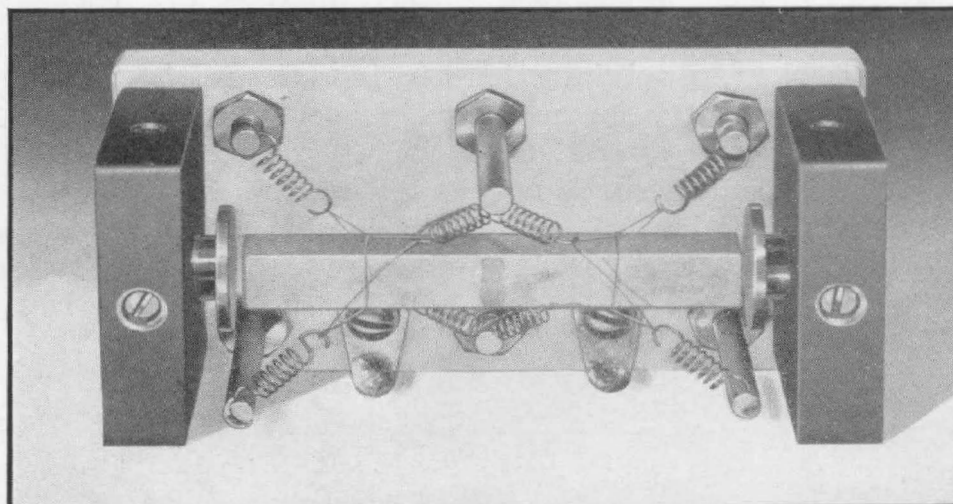
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THE NEW ELECTRICAL
UNITS 7

● **ONE OF THE** lowest frequency modes of vibration in quartz is that of a long thin bar vibrating in the direction of its length. This led to the early use of this mode in constant frequency oscillators and in filters where frequencies of the order of 30 to 100 kc were desired. The early bars were all X-cut, that is, the faces of the bar were

all parallel to the principal axes of the quartz crystal. Later developments have shown that rotation of the bar about the X-axis permits the choice of special properties with resulting improved performance in certain applications.

In the past, bars were generally operated at the fundamental frequency where the length of the bar is one-half wavelength. By choice of the lateral dimensions, the temperature coefficient of frequency could be reduced to a small value, averaging -0.75 parts per million per degree Centigrade for the TYPE 676-B bars. The use of adjustable baffles at the ends of the bar greatly reduced the damping caused by supersonic radiation and reduced changes in frequency caused by changes in air pressure. Since there was only one nodal region, at the center of the bar, mounting was somewhat difficult and there was a tendency for the bar to twist or displace in the mounting under shock.

Figure 1. View of the Type 1190 Quartz Bar with cover removed. The bar is suspended in nylon filaments held in tension by coiled springs.



In the TYPE 1190-A Quartz Bar,¹ operation at the second harmonic, with a full wavelength, provides two nodal regions so that a much more stable mechanical arrangement results. Furthermore, utilizing quartz of about the same lateral dimensions as before, zero temperature coefficient of frequency can be obtained for a desired temperature. Finally, in a tension mounting, the mounting conditions can be maintained essentially unchanged over long periods, resulting in less frequency drift with time.

From the piezoelectric relations and the equations of motion, the electrical impedance and the equivalent electrical circuit can be derived for a bar, either plated or unplated. Since practically all such bars are used with plated electrodes, we will consider only that case.

First we must realize that quartz is anisotropic, that is, its properties are different in different directions. Second, that there are certain axes of symmetry in quartz about which the properties repeat. For rotation about any one of the three *X*-axes, the properties repeat every 180 degrees. For rotation about the *Z*-axis, or optic axis, the properties repeat every 120 degrees. A familiar example of a simple anisotropic material is wood, where the properties are different along and across the grain.

The six stress-strain equations of a perfectly anisotropic material involve 36 elastic constants. Because of the reciprocity relationship, the number of independent constants reduces to 21. Next, applying the considerations of symmetry for rotations about any *X*-axis, in quartz, 8 of the 21 constants become zero. Finally, applying the consideration of symmetry for rotation about the *Z*-axis, one more constant

becomes zero and relationships are established between others such that the number of independent constants is reduced to six.

In general, additional stresses are introduced by the piezoelectric effect, so that the six general elastic equations are modified by additional terms involving 18 piezoelectric coefficients, which relate the stresses with the three component electric fields. Again applying symmetry conditions, the number of independent piezoelectric coefficients is reduced to two.

If we now consider an *X*-cut bar, with its length along the *Y*-axis, the piezoelectric equations reduce to:

$$- Y_y = s_{12} X_x + s_{11} Y_y + s_{13} Z_z \quad (1)$$

$$Q_x = E_x K / 4\pi - d_{11} X_x + d_{11} Y_y - d_{14} Y_z \quad (2)$$

where:

- Y_y = longitudinal strain = change in length per unit length = dl/l .
- X_x, Y_y, Z_z = The three longitudinal stresses = force per unit area = dynes/cm².
- Y_z = shearing stress = dynes/cm².
- d_{11}, d_{14} = piezoelectric constants = ESU charge/unit area/unit force = statcoulombs/cm²/dyne or strain/unit field = cms/statvolt
- E_x = applied field = voltage gradient = statvolts/cm.
- s_{11} , etc. = elastic compliances = displacement/dyne = cm²/dyne (These are the "zero field" values, sometimes written with a superscript *E*.)
- Q_x = charge per unit area = statcoulombs/cm².
- K = The "free" dielectric constant

To simplify these equations, we assume a bar having *X* and *Z* dimensions much smaller than the length, or *Y*, dimension, so that we can put $X_x = Z_z = Y_z = 0$ (3)

Equations (1) and (2) then become

$$-Y_y = s_{11} Y_y + d_{11} E_x \quad (4)$$

$$Q_x = E_x K / 4\pi + d_{11} Y_y \quad (5)$$

This obviously artificial simplification permits a solution to be obtained for the equivalent circuit. Since the length dimension *Y* is predominant, this theoretical solution will serve as a useful

¹Used in the new TYPE 1100-A Frequency Standards.



guide in practical cases. It would be expected that, as the lateral dimensions X and Z are increased, the departures from theory would become more pronounced. These departures are not so great as to limit the usefulness of the simplified interpretation until the lateral dimensions reach some 25 per cent of the length.

Equation (1) shows that, in addition to the desired lengthwise (Y) vibration, there will be vibrations in the directions of the width (Z) and thickness (X); also there will be a shearing vibration in the length-width plane (YZ). This condition may be thought of as a principal lengthwise vibration of the desired frequency having three other modes of vibration coupled to it. The effects of the other frequencies on the desired frequency will depend on the values of the frequencies (determined by dimensions and properties of the quartz in those directions) and on the magnitudes of the couplings between these different frequencies (determined by the properties of the quartz).

In general, if two modes of motion are in planes at right angles to each other, the coupling is weaker than if the motions are in the same plane. Also, if the modes of motion are such as to produce similar displacements at the boundaries, the coupling between them is much greater than when the boundary displacements are dissimilar.

In Equation (1), the frequencies of the thickness (X) and width (Z) longitudinal vibrations will be much higher than the desired length (Y) vibration. The frequency of the length-width (YZ) shear will decrease rapidly as the width is increased, approaching the length (Y) frequency, and the coupled effect becomes greater. Interpreted in this way, the simplification of Equation

(1) into Equation (4) is accomplished by making the coupled frequencies so high compared with the lengthwise (Y) frequency that their effects are negligible.

It can be shown that the admittance of the crystal is:

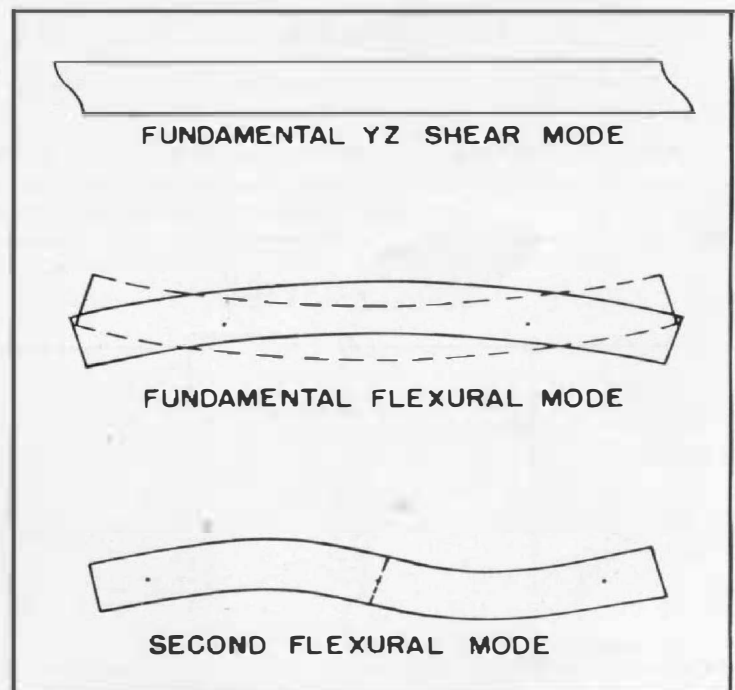
$$\frac{i}{E} = \frac{i}{E_x X} \quad (6)$$

$$= -j \frac{\omega E_x Y Z}{E_x X} \left\{ \frac{K}{4\pi} + \frac{d_{11}^2 \tan \frac{\omega Y}{2v}}{s_{11} \frac{\omega Y}{2v}} \right\}$$

where v is the velocity of wave propagation in quartz.

This consists of two terms, representing two branches in the equivalent circuit. The first represents the capacitance C_0 of the two electrodes separated by the quartz. The second term could be represented by the reactance of a transmission line. Usually, however, the impedance of this branch is represented by an inductance L_1 and capacitance C_1 in series, resonant at the resonant frequency of the quartz and having a

Figure 2. Secondary modes of vibration in the bar. The shearing vibration is excited by the normal electrode. The second flexural mode is mechanically excited by the similarity of displacement at the ends. The fundamental flexural mode cannot be excited by these electrodes or displacements.



reactance slope at this frequency equal to that of the tangent curve.

The inductance value can be written

$$L_1 = \rho \frac{XYZ}{2} \left[\frac{s_{11}}{2d_{11}Z} \right]^2 \times 9 \times 10^{11} \text{ henries} \quad (7)$$

The effective mass in the mechanical system, of a bar vibrating in the direction of its length, is one-half its actual mass. Consequently, in (7) the first factor represents the effective mass, or mechanical inductance, in the mechanical system. The second factor represents the transformation from the mechanical to the electrical system, and, to communications engineers, its representation as an electromechanical transformer with a certain transformation ratio is appealing.² The third factor is the conversion from electrostatic to practical units.

Similarly, for the capacitance:

$$C_1 = \frac{2}{\pi^2} \frac{Ys_{11}}{XZ} \left[\frac{2d_{11}Z}{s_{11}} \right]^2 \frac{1}{9 \times 10^{11}} \text{ farads} \quad (8)$$

On this basis, the equivalent circuit can be drawn as shown in Figure 3.

If the mechanical elements are "taken through" the electromechanical transformer, we have

$$L_1 = \frac{XY}{Z} 106.8 \text{ henries} \quad (9)$$

²"Electromechanical Transducers and Wave Filters," W. P. Mason, D. Van Nostrand Co., 1942.

$$C_1 = \frac{ZY}{X} 0.00322 \times 10^{-12} \text{ farads} \quad (10)$$

where the dimensions X, Y, Z are in cms.

Throughout the above no account has been taken of losses in the mechanical system. Due to factors such as supersonic radiation, dissipation in the mounting, and losses in the quartz, a resistance element should be included in the mechanical system.

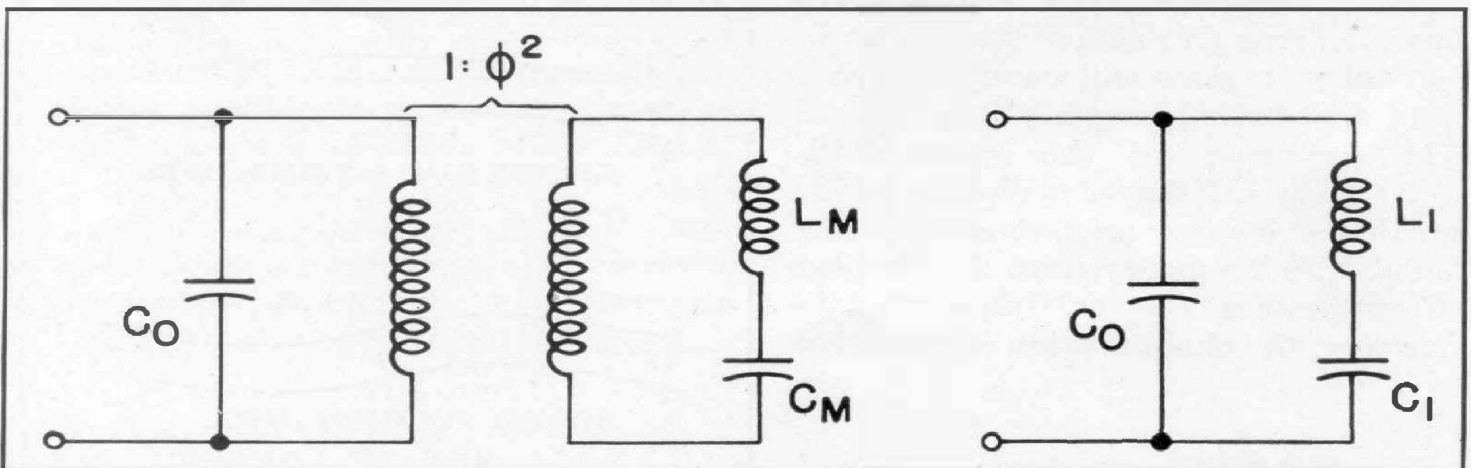
For a bar mounted in air, free of reflecting obstructions, the greatest energy loss is due to supersonic radiation from the ends of the bar. If the smallest dimension is greater than one-half wavelength in air, this loss can be calculated as follows:

At each end of the bar a mechanical resistance $R_m = (\rho v)_a XZ$ mechanical ohms is effective, where $(\rho v)_a$ is the specific radiation resistance of air and is equal to 43 mechanical ohms per square centimeter. In the mechanical system the two ends are in parallel, giving an effective resistance $R_m/2$. Taking this mechanical resistance through the electromechanical transformer, we obtain the electrical equivalent

$$R_{rad} = \frac{X}{Z} 6920 \text{ ohms} \quad (11)$$

The ratio of resonant reactance to

Figure 3. Equivalent circuit of the quartz bar. At the left is shown the equivalent circuit with the electromechanical transformation shown as a transformer of ratio $1:\phi^2$. If the mechanical quantities are "taken through" the transformer, the electrical equivalent circuit shown at the right results.





resistance, or Q , of the crystal can be determined either in the mechanical system or the electrical equivalent. In the mechanical system, the resonant mechanical reactance and the mechanical resistance are:

$$Q_m = \frac{X_m}{R_m} = \frac{\omega_R Y \rho X Z}{4(\rho v)_a X Z} \quad (12)$$

At resonance $\omega Y = \pi v_q$ so that

$$\begin{aligned} Q_m &= \frac{\pi(\rho v)_q}{4(\rho v)_a} \\ &= \frac{\pi(2.654 \times 5.44 \times 10^5)}{4 \times 43} \\ &= 26400 \end{aligned} \quad (13)$$

In the electrical equivalent circuit we have from Equations (9) and (11) the resonant reactance and resistance giving

$$Q_e = \frac{182.5 \times 10^6 X/Z}{6920 X/Z} = 26400 \quad (14)$$

Under these conditions it is seen that the Q is independent of the area of the ends of the bar, that is, a "thin" bar would have the same Q as a comparatively "fat" bar. However, operation under these conditions is of no practical interest since reflections would take place from objects near the crystal, and the losses are unnecessarily high.

One method of reducing the supersonic energy loss and of fixing the conditions of reflection near the crystal is to put baffle plates near the ends of the bar and adjust the position of the baffles for quarter-wave resonance.

For an X -cut 50-kc bar having the dimensions $X = 9.4$, $Y = 54.4$, $Z = 7.0$ mm, the resistance, when radiating freely, would be, from Equation (11), 9290 ohms. The actual measured resistance would be appreciably higher due to the effects of radiation from other surfaces, losses in the mounting, and the effects of coupled modes of motion.

Using properly adjusted baffles, the above radiation resistance can be re-

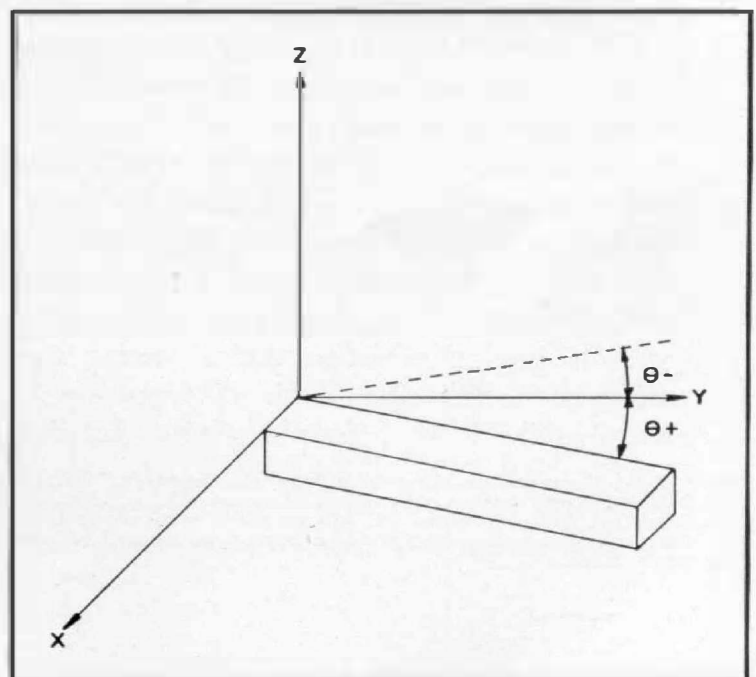
duced by 90 or 95 per cent, becoming about 700 ohms. The actual measured resistance under these conditions would be about 2500 ohms, giving a Q of 98,000.

These figures indicate that vacuum mounting of such a bar will not result in an indefinite increase in Q . Here, vacuum mounting could at most remove the residual air-wave loss at the ends (700 ohms) plus a small amount due to radiation from other surfaces — perhaps 800 ohms in all, in the total of 2500 ohms. This would give a Q of about $98,000 \times (2500/1700) = 147,000$.

The residual loss is largely due to coupled modes of motion. It can be substantially altered by orienting the bar. While this reduction would be highly desirable from the point of view of obtaining the greatest possible Q , other factors are important, such as increased temperature coefficient.

An unexpected effect occurs, as a result of the YZ shear vibration, which is the excitation of a flexural vibration. This flexural vibration cannot be ex-

Figure 4. Showing the relation of the rotation angle, Θ , to the principal axis of the quartz.



cited piezoelectrically with the simple electrode system so far considered. A mechanical coupling exists between the flexural and shear modes by virtue of similar displacements at the boundaries. This may be visualized as shown in Figure 2. It is evident that there is a marked similarity of displacements at the ends of the bar, for the fundamental shear and the second flexural mode.

The flexural frequency is given by km^2Z/Y^2 , where the factor m is a complicated function of the dimensions of the bar for all but extremely thin bars. An important property, however, is evident — that the flexural frequency *increases* with an *increase* in Z , or width, of the bar. This mode is the only one in which the frequency changes in this direction with change of dimension. When the width Z is made approximately 25 per cent of the length Y , the second flexural frequency is equal to the longitudinal frequency. Because of the comparatively strong coupling, the flexural frequency causes a marked disturbance of the longitudinal frequency and also causes a large increase in the temperature coefficient.

Further investigation of this region for bars having approximately square cross-section has shown that the coupled circuit effect may be utilized to obtain a frequency-temperature curve in the shape of an inverted parabola.³ At one temperature, the temperature-coefficient of frequency is zero, and changes only slowly for temperatures on either side.

So far we have indicated a reduction in the effects of coupled frequencies only by making the frequencies very far removed from the desired frequency, accomplished by keeping the lateral

dimensions of the bar very small compared with the length.

In Equation (1) it is possible to make certain of the elastic coefficients zero, by suitable orientation of the bar. Equation (1) is written with the particular values of the elastic and piezoelectric coefficients which apply when the three dimensions of the bar are taken along the three crystallographic axes.

Investigation of the properties of rotated bars shows that for a rotation of -18.5° , the coefficient s'_{24} vanishes. This means that for this rotation no shearing motion is excited and, consequently, no flexural motion will take place. Thus two of the disturbing frequencies are eliminated. The motion consists of the longitudinal vibration desired, accompanied by lateral motions only. A bar cut in this manner is found to have a higher Q than an unrotated bar. It has also a substantially higher temperature coefficient (approximately -24 parts per million per degree C. instead of -1 or -2), which is not too troublesome in some applications. At this angle the length of the bar is in the direction of minimum Young's Modulus in the YZ plane. For a given frequency this gives the shortest bar, some 6 per cent shorter than the unrotated bar.

At an angle of rotation of $+41.5$ degrees, not only is the coefficient s'_{24} zero, but also s'_{23} . The coefficient s'_{12} is much smaller than in the two previous cases. In this case a nearly pure longitudinal vibration is obtained, with a small lateral motion in the X -direction only.⁴ A substantially higher Q is obtained but at the expense of a high temperature coefficient, approximately -77 parts per million per degree C. At this angle the length of the bar is very

³"Low Temperature-Coefficient Quartz Crystals," W. P. Mason, *Bell System Technical Journal*, XIX, No. 1, January, 1940. On page 75, Mason gives the proportions for a square bar as 0.272. The bars here described are not square.

⁴"Some Experimental Studies of the Vibrations of Quartz Plates," R. B. Wright and D. M. Stuart, Research Paper No. 356, U. S. Bureau of Standards.

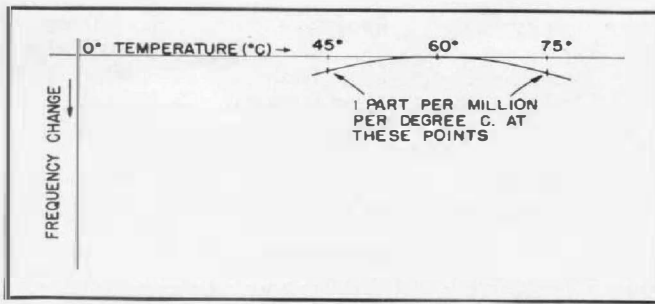


Figure 5. Plot of the change in frequency with the temperature for a Type 1190-A Quartz Bar.

nearly in the direction of maximum Young's Modulus in the YZ plane, giving very nearly the greatest length for a bar of given frequency, some 30 per cent longer than the unrotated bar.

The simple theory outlined above is not adequate to handle the case of bars the lateral dimensions of which are appreciable compared with the length.

In general, experiment shows that the Q of bars whose lateral dimensions are of the order of 25 per cent of the length is somewhat greater and that the temperature coefficient can be made zero at a particular temperature. The bar described above for 50-kc fundamental operation can be operated in the second harmonic mode at 100 kc, thereby bringing the ratio of dimensions into the range giving low temperature coefficient, without using more quartz. Because of the mode of vibration, two nodal regions are obtained instead of one, making for a much better mechanical mounting with support at two points. A value of Q of 170,000 can be obtained, in a thread-suspension mounting with baffles, of the type shown in the photograph of Figure 1.

— J. K. CLAPP

THE NEW ELECTRICAL UNITS

The change from the old "international" units to the new "absolute" units, which took place on January 1, 1948, was described under the above title in the *General Radio Experimenter* for July-August, 1947. This change involves a revaluation of the units of resistance, inductance, and capacitance as given in the following table:

1 international ohm	= 1.000495 abs. ohms
1 international henry	= 1.000495 abs. henries
1 international farad	= .999505 abs. farad

To convert the values of existing standards to the new units, the present values should be multiplied by these factors. To adjust an existing standard to have its marked value in the new units, a resistor or inductor is decreased in value by 0.0495 per cent, while a capacitor is increased in value by 0.0495 per cent.

All instruments now being manufactured, together with all of their components, are adjusted and calibrated in terms of the new units. The difference between the old and new units is of importance only when the accuracy limits are 0.1 per cent and 0.25 per cent. In this accuracy classification there are 12 types of General Radio instruments and components, as shown in the accompanying table. All of these sold after the approximate dates given in the table will be calibrated in absolute units and will be marked with the abbreviation "abs." placed after or near the unit in which the instrument is calibrated. The spread in these dates from February 15 to May 1 has been caused by the fact that orders for parts for a production lot of instruments must be placed many



months ahead of the final delivery to sales stock. It was necessary in such items as resistance units, which are manufactured in lots of several thousand and are used in almost all of this group of instruments, to arrange that all old "international" units be used up at about the same time and that new "absolute" units be then available.

New instruments with type numbers of 1000 and above and new components which may have type numbers below 1000 will not be marked "abs.", even though so calibrated.

Type	Resistors	Date
500	Resistor	April 15
510	Decade Resistance Unit	May 1
602	Decade Resistance Box	May 1
654	Decade Voltage Divider	May 1
668	Compensated Decade Resistance Unit	Feb. 15
670	Compensated Decade Resistor	Feb. 15
Capacitors		
722	Precision Condenser	April 15
509	Standard Condenser	March 15
Inductors		
106	Standard Inductance	March 15
Bridges		
716	Capacitance Bridge	March 15
821	Twin-T Impedance Measuring Circuit	May 1
667	Inductance Bridge	March 1

READJUSTMENT OF OLD INSTRUMENTS

The difference of approximately 0.05 per cent between the old and the new units will be of little importance for much ordinary measurements work. On the other hand, in measurements where an accuracy approaching 0.1 per cent is desired, allowance for the difference

between the two systems of units should be made. It is possible to readjust or recalibrate bridges and impedance standards in the absolute system of units, and our Service Department will quote prices upon request.

— ROBERT F. FIELD

MISCELLANY

Dr. M. A. El Said, Senior Lecturer at Fouad I University, Cairo, Egypt, at the request of the General Radio Company, visited the plant during January and February to demonstrate new electronic circuits particularly adapted to obtaining products and quotients of electrical voltages. One of these circuits is now under development for use in an electronic wattmeter.

The quartz bar illustrated in the article by J. K. Clapp is used in the new General Radio TYPE 1100-A Frequency Standard. Two models of this standard, the TYPE 1100-AP Primary Standard and the TYPE 1100-AQ Secondary Standard, are now in production and will be described in an early issue of the *Experimenter*.

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